Novel Method for Calculation and Measurement of Unloaded Q-Factor of Superconducting Dielectric Resonators

Mohan V. Jacob[a], Janina Mazierska[a], Kenneth Leong[a] and Jerzy Krupka[b]

[a]Electrical and Computer Engineering, James Cook University, Townsville, QLD 4811, AUSTRALIA

[b]Instytut Mikroelektroniki i Optoelektroniki Politechniki Warszawskiej, Koszykowa 75, 00-662 Warszawa, POLAND

Abstract — **The dielectric resonator technique is recognised as the best method for the measurement of surface resistance (R_s) of High Temperature Superconducting thin films. The R_s is calculated from the Unloaded Q-factor (Q_0) of the resonator, and to obtain accurate values of the Q_o-factor multi-frequency measurements of S_{21}, S_{11} and S_{22} and data circle fitting are required. As a result, surface resistance measurements at varying temperatures are very time consuming. In this paper we introduce a simplified method for calculations the Q_0-factor, which require measurements of S_{11} and S_{22} at the lowest temperature only. For all other temperatures only S_{21} measurements are needed. The method has shown to give sufficiently accurate Q_0 values and hence the surface resistance.**

I. Introduction

The quality of High Temperature Superconductors (HTS) at microwave frequencies is assessed by measuring the surface resistance of the materials. As it is well known the surface resistance cannot be measured directly and is found from the loss equation of a resonating structure through the S-parameters' measurements. From the measured S parameters one can calculate the unloaded Q_0 factor and hence the surface resistance of a material under test. The accuracy of surface resistance measurements depends strongly on the accuracy of unloaded Q-factor measurements. Different resonators (e.g. cavity, microstrip, parallel plate, confocal and dielectric rod resonators) have been employed for measurements of surface resistance of High Temperature Superconductors [1] with the last one providing high accuracy and sensitivity of measurements. There are two types of dielectric resonators used for HTS microwave characterisation: Hakki-Coleman (H-C) and Open Ended resonators [1]. The H-C structure consist of a dielectric rod sandwiched in between two superconducting samples enclosed in a metallic cavity which is schematically presented in Figure 1. When a low loss dielectric rod is employed the total loss of the resonator is mostly determined by the loss in the superconducting material. The Hakki-Coleman type of resonator is currently under consideration as a standard test fixture for microwave characterisation of HTS materials by the International Electro-technical Commission (IEC) [2].

For a very low coupling of the resonator, the unloaded Q_0-factor can be assumed to be equal to the loaded Q_L-factor, and in such a case measurements of S_{21} (or S_{11}) only are sufficient. However it may not be practical to perform measurements at very week coupling. For stronger coupling, providing the coupling at port 1 and 2 of the resonator are same, the Q_0 can be calculated using the formula [3]:

$$Q_0 = \frac{Q_L}{1-|S_{21}|} \qquad (1)$$

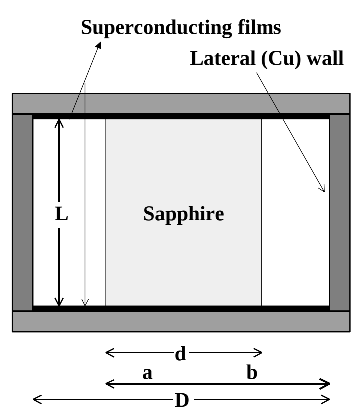

Figure 1 The schematic of the dielectric resonator.

In practice it is difficult to obtain equal coupling on both ports and hence the accuracy of the above equation may not be sufficient. Hence to calculate the Q_0-factor accurately under arbitrary coupling the following equation

[3] should be used:

$$Q_0 = Q_L(1 + k_1 + k_2) \quad (2)$$

where k_1 and k_2 are coupling coefficients

$$k_1 = \frac{1-|S_{11}|}{|S_{11}|-|S_{22}|} \quad \text{and} \quad k_2 = \frac{1-|S_{22}|}{|S_{11}|-|S_{22}|}$$

To use equation (2) for accurate calculations of the Q_0-factor multi-frequency measurements of S_{21}, S_{11} and S_{22} are needed. Measurements of surface resistance of HTS films at different temperatures and power levels require very long time for measurements and data processing. In this paper, we have introduced a novel, less time consuming method but sufficiently accurate for calculations of the Q_0-factor.

II. Novel method for Q_O-factor calculations at varying temperatures

For a dielectric resonator, the Unloaded Q-factor can be calculated using Eq. (2). Now let us consider two S-parameter measurements of a dielectric resonator at two different temperatures T_A and T_B. At temperature T_A, the unloaded Q_{0A}-factor is:

$$Q_{0A} = Q_{LA}(1 + k_{1A} + k_{2A}) \quad (3)$$

and the coupling coefficients k_{1A} and k_{2A} are described as:

$$k_{1A} = \frac{Q_{0A}}{Q_{ext1A}} \quad (4a)$$

$$k_{2A} = \frac{Q_{0A}}{Q_{ext2A}} \quad (4b)$$

At temperature T_B the unloaded Q_{0B} -factor is,

$$Q_{0B} = Q_{LB}(1 + k_{1B} + k_{2B}) \quad (5)$$

and coupling coefficients can be described as:

$$k_{1B} = \frac{Q_{0B}}{Q_{ext1B}} \quad (5a)$$

$$k_{2B} = \frac{Q_{0B}}{Q_{ext2B}} \quad (5b)$$

If we assume that the external Q-factors (Q_{ext}) at both temperatures T_A and T_B are the same, i.e., $Q_{ext1A} = Q_{ext1B}$ and $Q_{ext2A} = Q_{ext2B}$ then the Eq. (5) can be rewritten as:

$$Q_{0B} = Q_{LB}\left[1 + \frac{Q_{0B}}{Q_{ext1B}} + \frac{Q_{0B}}{Q_{ext2B}}\right] \quad (6)$$

$$Q_{0B} = Q_{LB}\left[1 + Q_{0B}\left(\frac{1}{Q_{ext1B}} + \frac{1}{Q_{ext2B}}\right)\right] \quad (7)$$

Similarly,

$$Q_{0A} = Q_{LA}\left[1 + Q_{0A}\left(\frac{1}{Q_{ext1A}} + \frac{1}{Q_{ext2A}}\right)\right] \quad (8)$$

Equation (8) can be re-written as:

$$\frac{\frac{Q_{0A}}{Q_{LA}} - 1}{Q_{0A}} = \frac{1}{Q_{ext1A}} + \frac{1}{Q_{ext2A}} \quad (9)$$

Using Eq. (9) in Eq. (7) the following expressions for the unloaded Q_{0B} -factor at temperature T_B are obtained:

$$Q_{0B} = \frac{Q_{LB}}{1 - \frac{Q_{LB}}{Q_{LA}}\left(\frac{Q_{0A}}{Q_{LA}} - 1\right)} \quad (10)$$

$$\text{or} \quad Q_{0B} = \frac{Q_{LB}}{1 - \frac{Q_{LB}}{Q_{0A}}\left(k_{1A} + k_{2A}\right)} \quad (11)$$

The equation (10) or (11) can be used for set of j temperatures to obtain a characteristic of Q_{0j} versus temperature.

The assumption of the external Q-factors being constant with temperature varying can be considered correct if parameters of the external circuity do not vary with temperature of the resonator. This requires the reference plane of the coupling coefficient to be defined at the terminals of a cryostat in which the resonator is housed (Figure 2). Hence this implies that cables inside the cryostat are considered as a part of the resonator and a special data processing needs to be used to eliminate their influence from measured parameters [4] to avoid errors.

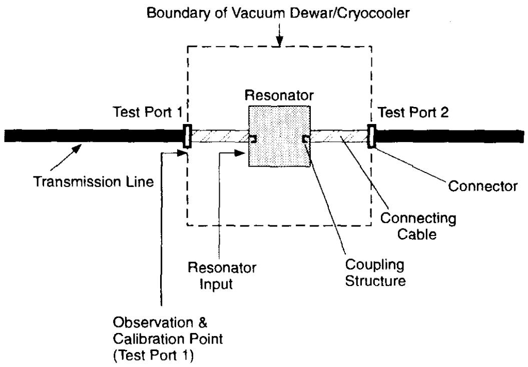

Figure 2 The reference plane of the resonator and the position of coupling loops

III. VERIFICATION OF THE PROPOSED Q_0 MEASUREMENT METHOD

To verify the presented method for simplified but accurate measurements of surface resistance of High Temperature Superconductors at varying temperatures we performed measurements of high quality $YBa_2Cu_3O_{7-\delta}$ (YBCO) thin films. Figure 3 presented the measurement system. The resonant frequency of the dielectric resonator is 10 GHz.

In the verification process we measured S_{21}, S_{11} and S_{22} parameters at differing temperatures from 28K to 85K. Then two calculation procedures have been applied. In the first procedure the equation (2) has been used for calculations of the unloaded Q_0-factor at each temperature. The loaded Q_L-factor was calculated from multi-frequency measurements of S_{21} and the coupling coefficients k_1 and k_2 from multi-frequency measurements of S_{11} and S_{22} using the Transmission Mode Q-factor (TMQF) Technique [4]. The TMQF method takes into account various possible losses and errors such as noise, cross-talk, coupling loss, coupling reactance and electrical delay due to uncalibrated transmission lines of real measurement systems. The uncertainty in Q_0 is expected to be less than 0.1% for Q_0 factors above 100,000.

In the second procedure we used Eq. (11) for calculations of the unloaded Q_0 -factor. The measurements of S_{21} were used for calculations of Q_{LB} and Q_{0A}, k_{1A} and k_{2A} were constants obtained from measurements at temperature of 28K using the TMQF technique. The average surface resistance of the YBCO thin films was calculated for both procedures from the loss equation of the resonator.

$$R_{SS} = A_S\left(\frac{1}{Q_0} - \frac{R_{SM}}{A_M} - r_e \tan d\right) \quad (19)$$

where, A_S and A_M are the geometric factors of the endwalls (Superconducting) and the lateral wall (Copper), R_{SM} is the surface resistance of copper, ρ_e is the energy filling factor and $\tan\delta$ is the loss tangent of the dielectric (sapphire).

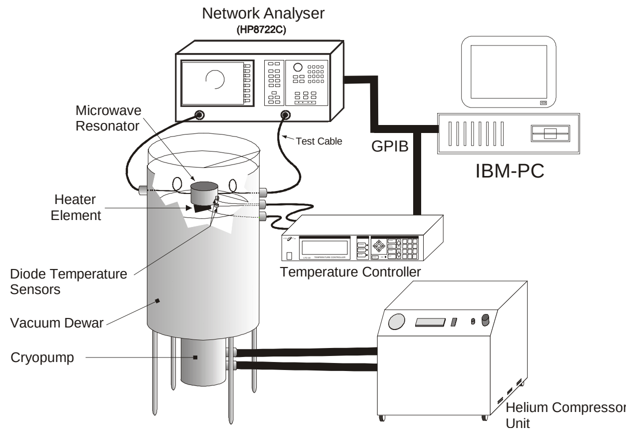

Figure 3 The measurement system

IV. DISCUSSION OF THE Q_0 FACTORS AT VARYING TEMPERATURES OBTAINED USING THE FULL AND THE SIMPLIFIED PROCEDURES

Two procedures to calculate Q_0-factor have been compared on the basis of measurements of the high quality YBCO sapphire resonator at frequency of 10 GHz and temperature range from 28K to 85K as described in Section III of this paper. Figure 4 presents measured coupling coefficients obtained from multi-frequency measurements of S11 and S22 parameters and the Transmission Mode Q-factor Technique.

Figure 5 shows the Q_0-factor values calculated using the accurate and the method presented in this paper. Figure 6 presents the surface resistance of the YBCO thin film under test calculated using the Q_0-factor obtained using the accurate and the simplified method. The agreement between both the methods is very good especially when saving the time is concerned. To carry multi-frequency measurements of S_{11}, S_{21} and S_{22} at all temperatures (25 - 85K at an interval of 2K) for the accurate method approximately 20 hours to complete the measurements was required. It took around 6 MB of hard disk space to store the measurement data. To perform measurements of the S_{11}, S_{21} and S_{22} parameters at one temperature and S_{21} measurements only for all other temperatures, the total time required was 12 hours and hard disk space required was reduced to 2 MB.

To carry out the data processing for all S-parameters using the TMQF technique for all temperatures it took approximately 3 hours while the simplified method required less than an hour. Therefore using the new technique we were able to save about 8 hours and 4 MB of hard disk space per measurement without compromising the accuracy of results.

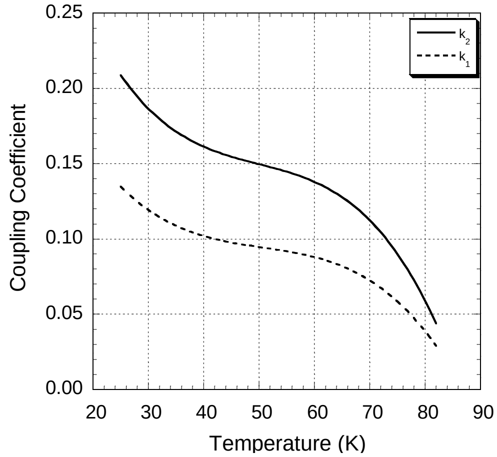

Figure 4 Coupling Coefficients at port 1 (k_1) and port 2 (k_2) at differing temperatures.

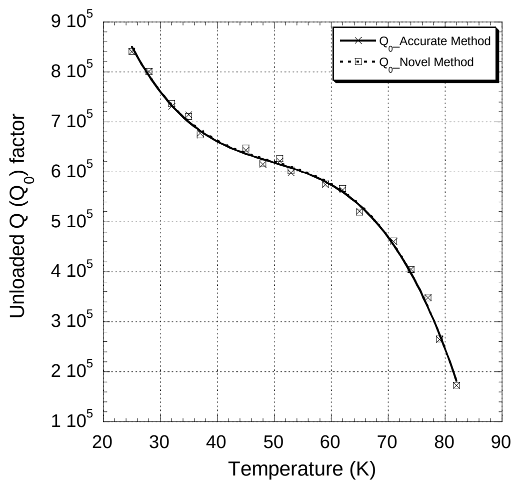

Figure 5 The calculated unloaded Q-factor using the accurate and simplified methods.

V. CONCLUSION

We presented a simplified novel method for measurements and calculations of the unloaded Q-factor for varying temperatures. This method can be useful for measurements of surface resistance of High Temperature Superconductors. The developed method has proved to provide accurate results, with maximum difference of 1% as compared to the accurate TMQF technique. The novel method allows for reduction of number of measurements data, measurement time and the computer memory.

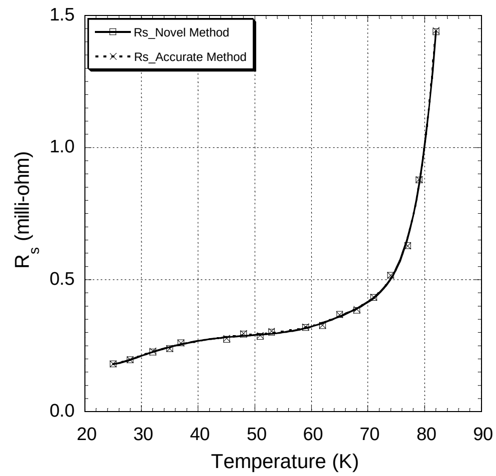

Figure 6 The calculated surface resistance of YBCO thin film using the accurate and simplified methods.

VI. ACKNOWLEDGEMENTS

This work was financially supported by ARC-Large grant, Australia. One of the authors (MVJ) acknowledges the JCU Post Doctoral Fellowship.

VII. REFERENCES

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